IFUP-TH 2002-26

Gravitational deflection of light and helicity asymmetry

E. GUADAGNINI

Dipartimento di Fisica *Enrico Fermi* dell'Università di Pisa
and INFN Sezione di Pisa
Via F. Buonarroti, 2. 56100 - PISA - Italy

Abstract. The helicity modification of light polarization which is induced by the gravitational deflection from a classical heavy rotating body, like a star or a planet, is considered. The expression of the helicity asymmetry is derived; this asymmetry signals the gravitationally induced spin transfer from the rotating body to the scattered photons.

E-Mail: guada@df.unipi.it

1. The asymmetry. Gyroscopic effects in gravity, like the Lens-Thirring [1] and the Skrotskii effect [2] have been the subject of considerable research [3,4]. In this article I shall consider a related issue; namely, the helicity modification of the light polarization which is induced by the gravitational deflection from a classical heavy rotating source, like a star or a planet. The transition amplitude of the gravitational scattering for the different polarization states of the photons will be computed. It turns out that, because of the nonvanishing angular momentum of the classical source, the transition probabilities which are associated with the two helicity states of the deflected photons may differ. As a result, even if the incoming radiation is not polarized, the deflected photons may have a nontrivial elliptic polarization corresponding to a nonvanishing total intrinsic spin. In order to illustrate this phenomenon, let us consider for instance the light deflected from the Sun. As shown in Figure 1, suppose that the incoming flux of unpolarized photons is directed as the angular momentum vector of the Sun.

Figure 1. Light deflection.

Let $n_+(\theta)$ $(n_-(\theta))$ be the number of deflected photons with helicity $+1$ (-1) at the scattering angle θ. Then, the prediction for the helicity asymmetry $\chi(\theta)$ is

$$\chi(\theta) \equiv \frac{n_+(\theta) - n_-(\theta)}{n_+(\theta) + n_-(\theta)} \simeq 2\pi \frac{J\,\theta^2}{M\,c\,\lambda} \quad , \tag{1}$$

where J denotes the magnitude of the angular momentum of the Sun, M represents its mass, and λ is the wave length of the electromagnetic radiation. By inserting the values $J \simeq 10^{40}$ kg m^2/s , $\theta \simeq 4 \times 10^{-6}$, $\lambda \simeq 5 \times 10^{-7}$ m , one finds

$$\chi \simeq 0.33\ \% \quad , \tag{2}$$

which appears to be suitable for an experimental verification. The derivation of equation (1) and a few comments on its possible applications are in order.

2. The computation. Let us consider effective quantum gravity [5,6], which provides the natural quantum field theory interpretation of Einstein's theory of gravitation. Effective quantum gravity is a phenomenological theory which can be understood as the gravitational analogue of the Fermi theory of the beta decay for weak interactions. The spacetime metric $g_{\mu\nu}(x)$ of general relativity is written as $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$, where $\eta_{\mu\nu}$ represents the Minkowski metric of flat spacetime; in cartesian coordinates $\{ x^\mu = (x^0, \vec{x}) \}$, $\eta_{\mu\nu}$ takes the standard diagonal form $\text{diag.}(\eta_{\mu\nu}) = (+1, -1, -1, -1)$. The effective quantum field theory description of gravity is based on a perturbative expansion in powers of $h_{\mu\nu}$. In the following computation, the standard conventions $\hbar = c = 1$ will be adopted.

• **Gravitational couplings.** In the large distance limit and to first order in v/c, the coupling of the fluctuation field $h_{\mu\nu}(x)$ with a classical heavy rotating body, which is subject to stationary conditions and is placed in position $\vec{r}$, is given by the action term

$$\begin{aligned} S_{(1)} &= -\tfrac{1}{2} \int d^4x \, \Theta^{\mu\nu}(\vec{x}\,) \, h_{\mu\nu}(x) \\ &= -\tfrac{1}{2} \int d^4x \, \left[M \, h_{00}(x) + \epsilon^{ijk} \, J_i \, \partial_j h_{0k}(x) \right] \, \delta^3(\vec{x} - \vec{r}\,) \quad , \end{aligned} \tag{3}$$

where M denotes the mass of the body and $\{ J_i \}$ are the components of its total angular momentum. The interaction of the metric fluctuation $h_{\mu\nu}(x)$ with photons is described by the energy-momentum coupling

$$S_{(2)} = -\tfrac{1}{2} \int d^4y \, T^{\mu\nu}(y) \, h_{\mu\nu}(y) \quad , \tag{4}$$

with

$$T^{\mu\nu}(y) = F^{\mu\sigma}(y) \, F_\sigma{}^\nu(y) + \tfrac{1}{4} \eta^{\mu\nu} \, F_{\lambda\sigma}(y) \, F^{\lambda\sigma}(y) \quad . \tag{5}$$

• **Gravitational scattering.** The gravitational scattering of one photon from a classical heavy rotating body is quite similar to the Rutherford scattering of one electron from a fixed Coulomb potential in electrodynamics. Suppose that, in the initial state, the photon has momentum $\vec{p}$ and polarization α; $|\, \text{in} \,\rangle = |\, \vec{p}, \alpha \rangle = a^+(\vec{p}, \alpha) \, |0\rangle$ where $a^+(\vec{p}, \alpha)$ is the creation operator of the photon. Let $\vec{k}$ and β be the final momentum and polarization of the photon. At the tree-level, the amplitude A of this particular "newtonian scattering" is given by

$$A = \tfrac{i^2}{4} \int d^4x \, d^4y \, \Theta^{\lambda\sigma}(\vec{x}\,) \, \overline{h_{\lambda\sigma}(x) \, h}_{\mu\nu}(y) \, \langle \vec{k}, \beta \,|\, T^{\mu\nu}(y) \,|\, \vec{p}, \alpha \rangle \quad . \tag{6}$$

By using the Einstein-Hilbert action for the metric field $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$, one can derive the expression of the graviton propagator which, in Feynman gauge, takes the form

$$\overline{h^{\mu\nu}(x) \, h}_{\tau\sigma}(y) = i \, (16\pi G) \int \frac{d^4p}{(2\pi)^4} \, \frac{e^{-ip(x-y)}}{p^2 + i\,\epsilon} \, (\delta^\mu_\tau \, \delta^\nu_\sigma + \delta^\mu_\sigma \, \delta^\nu_\tau - \eta^{\mu\nu} \, \eta_{\tau\sigma}) \quad . \tag{7}$$

• **Transition amplitude.** Because of the invariance under time translations, energy is conserved and then $|\vec{k}\,| = |\vec{p}\,| = p$. The momentum transfer of the diffusion process is described by the vector $(\vec{k} - \vec{p}\,)$ which has magnitude $|\vec{k} - \vec{p}\,| = 2p \sin\theta/2$ where θ denotes the scattering angle. Let $\vec{\varepsilon}_\alpha$ and $\vec{\varepsilon}_\beta$ be the polarization vectors of the incoming and outcoming photon respectively. The transition amplitude (6) can be written as

$$A \;=\; \frac{i\,G\,M\,\delta(|\vec{k}\,| - |\vec{p}\,|)}{\pi\, p\, |\vec{k} - \vec{p}\,|^2}\,(B_1 \,+\, B_2) \quad , \tag{8}$$

where

$$B_1 \;=\; \tfrac{1}{2}|\vec{k} + \vec{p}\,|^2 \big(\vec{\varepsilon}^{\,*}_\beta \cdot \vec{\varepsilon}_\alpha\big) \;-\; \big(\vec{p} \cdot \vec{\varepsilon}^{\,*}_\beta\big)\big(\vec{k} \cdot \vec{\varepsilon}_\alpha\big) \quad , \tag{9}$$

$$B_2 \;=\; \frac{i\,p}{M} \epsilon^{ijk}\, J_i \left(\vec{k} - \vec{p}\,\right)_j \Big[\big(\vec{k} + \vec{p}\,\big)_k \big(\vec{\varepsilon}^{\,*}_\beta \cdot \vec{\varepsilon}_\alpha\big) \;-\; (\vec{\varepsilon}_\alpha)_k \big(\vec{p} \cdot \vec{\varepsilon}^{\,*}_\beta\big) \;-\; (\vec{\varepsilon}^{\,*}_\beta)_k \big(\vec{k} \cdot \vec{\varepsilon}_\alpha\big) \Big] \quad . \tag{10}$$

Let us introduce a basis of linear polarizations for the photons. The scattering plane can be identified with the xy-plane; more precisely, the components of the initial and final momenta are taken to be $\vec{p} = (p, 0, 0)$ and $\vec{k} = (p\cos\theta, p\sin\theta, 0)$. Thus, one can put

$$\vec{\varepsilon}_\alpha(1) \;=\; (0,1,0)\;,\; \vec{\varepsilon}_\alpha(2) \;=\; (0,0,1)\;,\; \vec{\varepsilon}_\beta(1) \;=\; (-\sin\theta, \cos\theta, 0)\;,\; \vec{\varepsilon}_\beta(2) \;=\; (0,0,1)\;.$$

The amplitude can then be written in matrix form with respect to the polarization states

$$\begin{bmatrix} (1)_\alpha \otimes (1)_\beta & (1)_\alpha \otimes (2)_\beta \\ (2)_\alpha \otimes (1)_\beta & (2)_\alpha \otimes (2)_\beta \end{bmatrix} \quad .$$

One finds

$$A \;=\; \frac{i\,G\,M\,\delta(|\vec{k}\,| - |\vec{p}\,|)\,\cos\theta/2}{2\,\pi\,p\,\sin^2\theta/2} \begin{bmatrix} \cos\theta/2 - i\frac{2pJ_3}{M}\sin\theta/2 & -i\frac{2p\widetilde{J}}{M}\sin^2\theta/2 \\ i\frac{2p\widetilde{J}}{M}\sin^2\theta/2 & \cos\theta/2 - i\frac{2pJ_3}{M}\sin\theta/2 \end{bmatrix} \tag{11}$$

where J_3 is the component of the angular momentum of the source which is orthogonal to the scattering plane whereas $\widetilde{J}$, given by

$$\widetilde{J} \;=\; J_1\,\cos\theta/2 \,+\, J_2\,\sin\theta/2 \quad , \tag{12}$$

is the component of the angular momentum which belongs to the scattering plane and is orthogonal to $(\vec{k} - \vec{p}\,)$. Note that the J_3-contribution to the amplitude is proportional to the identity matrix; so, the component J_3 does not modify the polarization state of the photons.

Now, the newtonian scattering we wish to analyze is similar but not equal to the Coulomb scattering of electrons. In the gravitational deflection of light from a star or a planet, the typical wavelength of the photon is very small if compared to the distance of

the photon from the source or the size of the source. In practice, the wave packet associated with the orbital state of the photon moves in an "almost constant" gravitational potential. In the limiting case of an "exactly constant" external potential, the momentum transfer would vanish. Consequently, in order to extract from expression (11) the part $A_{\rm rel}$ of the transition amplitude which is relevant for light deflection, one must consider the $(\vec{k}-\vec{p}) \to 0$ limit. The vector $(\vec{k}-\vec{p})$ has magnitude $|\vec{k}-\vec{p}| = 2p \sin\theta/2$; since energy is fixed, p is fixed and thus the $(\vec{k}-\vec{p}) \to 0$ limit is equivalent to the $\theta \to 0$ limit. The leading term of each matrix element of the amplitude (11) in the formal $\theta \to 0$ limit gives

$$A_{\rm rel} \;=\; \frac{i\, 4\, G\, M\, \delta(|\vec{k}\,| - |\vec{p}\,|)}{2\,\pi\, p\, \theta^2} \begin{bmatrix} 1 & -i\, J_1\, p\, \theta^2/2M \\ i\, J_1\, p\, \theta^2/2M & 1 \end{bmatrix} \quad . \tag{13}$$

It will now be assumed that expression (13) represents the relevant transition amplitude for the gravitational deflection of light. The component J_1 of angular momentum which enters equation (13) simply denotes the component of angular momentum which is parallel to the incoming momentum $\vec{p}$. Thus, one really has $J_1 = (\vec{J}\cdot\vec{p}\,)\,/\,|\vec{p}\,|$.

As a first check, let us consider the differential cross section; in the $\theta \to 0$ limit, equation (13) implies

$$\frac{d\sigma}{d\Omega} \;=\; \frac{16\, G^2\, M^2}{\theta^4} \quad . \tag{14}$$

On the other hand, in the semiclassical limit the cross section takes the form

$$\frac{d\sigma}{d\Omega} \;\simeq\; \frac{b}{\theta} \left| \frac{db}{d\theta} \right| \quad , \tag{15}$$

where b is the impact parameter. By comparing expressions (14) and (15), one finds

$$\theta \;\simeq\; \frac{4\, G\, M}{b} \quad , \tag{16}$$

which is in agreement with the prediction based on classical arguments [7].

• **Helicity states.** Let us now concentrate on photon polarizations. According to the result (13), if the polarization state of the incoming photon is described by $\vec{\varepsilon}_\alpha(1) = (0,1,0)$ then, in the $\theta \to 0$ limit, the polarization vector $\vec{\varepsilon}$ of the deflected photon is

$$\vec{\varepsilon} \;\simeq\; \vec{\varepsilon}_\beta(1) \,+\, i\left(\vec{J}\cdot\vec{p}\;\theta^2/2M\right)\, \vec{\varepsilon}_\beta(2) \quad , \tag{17}$$

which corresponds to elliptic polarization. This phenomenon differs from the Skrotskii effect which only concerns the rotation angle of the transported electric field direction. For the outcoming photons, let us introduce the polarization vectors $\vec{\varepsilon}_\beta(\pm)$ corresponding to polarization states with ± 1 helicity

$$\vec{\varepsilon}_\beta(\pm) \;=\; \frac{1}{\sqrt{2}}\,(\,\vec{\varepsilon}_\beta(1) \,\pm\, i\,\vec{\varepsilon}_\beta(2)\,) \quad . \tag{18}$$

Then relation (17) takes the form

$$\vec{\varepsilon} \simeq \frac{1}{\sqrt{2}} \left[1 + \left(\vec{J} \cdot \vec{p} \, \theta^2 / 2M \right) \right] \vec{\varepsilon}_\beta(+) + \frac{1}{\sqrt{2}} \left[1 - \left(\vec{J} \cdot \vec{p} \, \theta^2 / 2M \right) \right] \vec{\varepsilon}_\beta(-) \quad . \tag{19}$$

Assuming $\left(\vec{J} \cdot \vec{p} \, \theta^2 / 2M \right) \ll 1$, the probability $w(\pm)$ of detecting a scattered photon with helicity ± 1 is

$$w(\pm) \simeq \tfrac{1}{2} \pm \left(\vec{J} \cdot \vec{p} \, \theta^2 / 2M \right) \quad . \tag{20}$$

Consequently, when the initial polarization is described by $\vec{\varepsilon}_\alpha(1)$, one has

$$w(+) - w(-) \simeq \left(\vec{J} \cdot \vec{p} \, \theta^2 / M \right) \quad . \tag{21}$$

It is easy to verify that, when the initial polarization is described by $\vec{\varepsilon}_\alpha(2) = (0,0,1)$, equation (21) is still valid. This concludes the derivation of the helicity asymmetry expression (1) for the gravitational deflection of light.

3. Gravitational spin transfer. The helicity asymmetry is originated by a nontrivial angular momentum of the deflecting body. A simple argument explains the structure of expression (1). Consider the coupling (3) of the classical source with the metric fluctuation $h_{\mu\nu}$. With respect to the gravitational mass term $M h_{00}$, which does not induce helicity modifications, the angular momentum term $\epsilon^{ijk} J_i \partial_j h_{0k}$ has strength J/M times the magnitude $p\,\theta$ of the momentum transfer which corresponds to the spatial derivative acting on h_{0k}. In standard $\hbar = c = 1$ units, the value p of the momentum of the photon equals $2\pi/\lambda$. Consequently, the non-diagonal matrix elements of the transition amplitude must be of order $2\pi J\theta/M\lambda$ (at least) with respect to the diagonal elements. The actual computation shows that non-diagonal matrix elements contain an extra multiplicative θ factor. Because of the spatial derivative acting on h_{0k}, the contribution of the lagrangian term $\epsilon^{ijk} J_i \partial_j h_{0k}$ to the transition amplitude must contain the imaginary unit $i = \sqrt{-1}$ with respect to the remaining part, and this implies the occurrence of helicity flips during the scattering. Finally, the sign of the asymmetry is fixed by consistency; in agreement with the results of the computations, the spin excess of the deflected photons must be directed as the angular momentum vector of the source.

The phenomenon associated with equation (1) can be understood as a gravitationally induced spin transfer from a classical rotating body to the scattered photons. Even if expression (1) does not contain $\hbar$, it is not obvious how to explain this spin transfer in purely classical terms. Formula (1) has been obtained by using the simplest modelling of the gravitational scattering and can be improved by taking into account the particular details of real macroscopic bodies. Finally, it is important to note that the asymmetry (1) corresponds to the vacuum optical activity; in practice, ordinary electromagnetic effects must also be considered.

In principle, the gravitationally induced spin transfer from a classical rotating body to photons appears to be one of the simplest tests of the existence of gyroscopic effects in gravity. This does not necessarily mean that the actual experimental verification of this phenomenon will be an easy task. For example, electromagnetic contributions to the helicity asymmetry should be clearly identified. In the context of gravitational lensing, the gravitational helicity asymmetry could be used to determine the magnitude of the angular momentum of rotating galaxies.

Aspects of the light propagation in various spacetimes have been discussed for instance by L. Blanchet, S. Kopeikin and G. Schafer, M.P. Haung and C. Lammerzahl, and Wei-Tou Ni in ref.[4]; additional material can be found in [8,-,11] and referenced quoted therein. Recently, classical gravitomagnetic effects have been considered also in [12,13,14].

I wish to thank Steven Shore for useful discussions.

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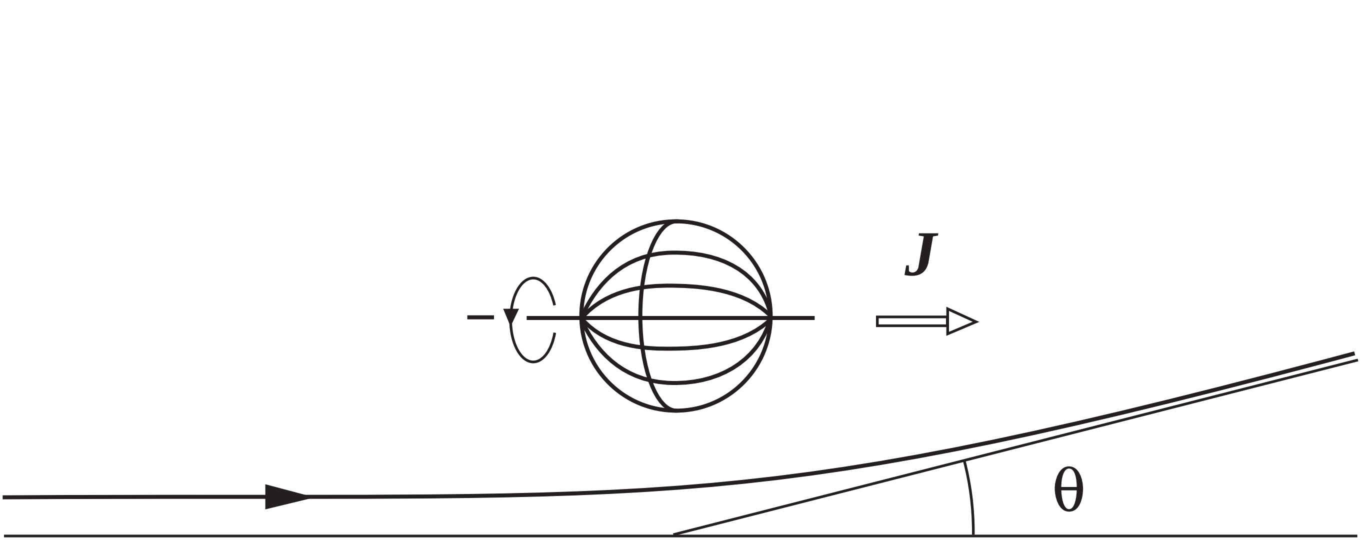
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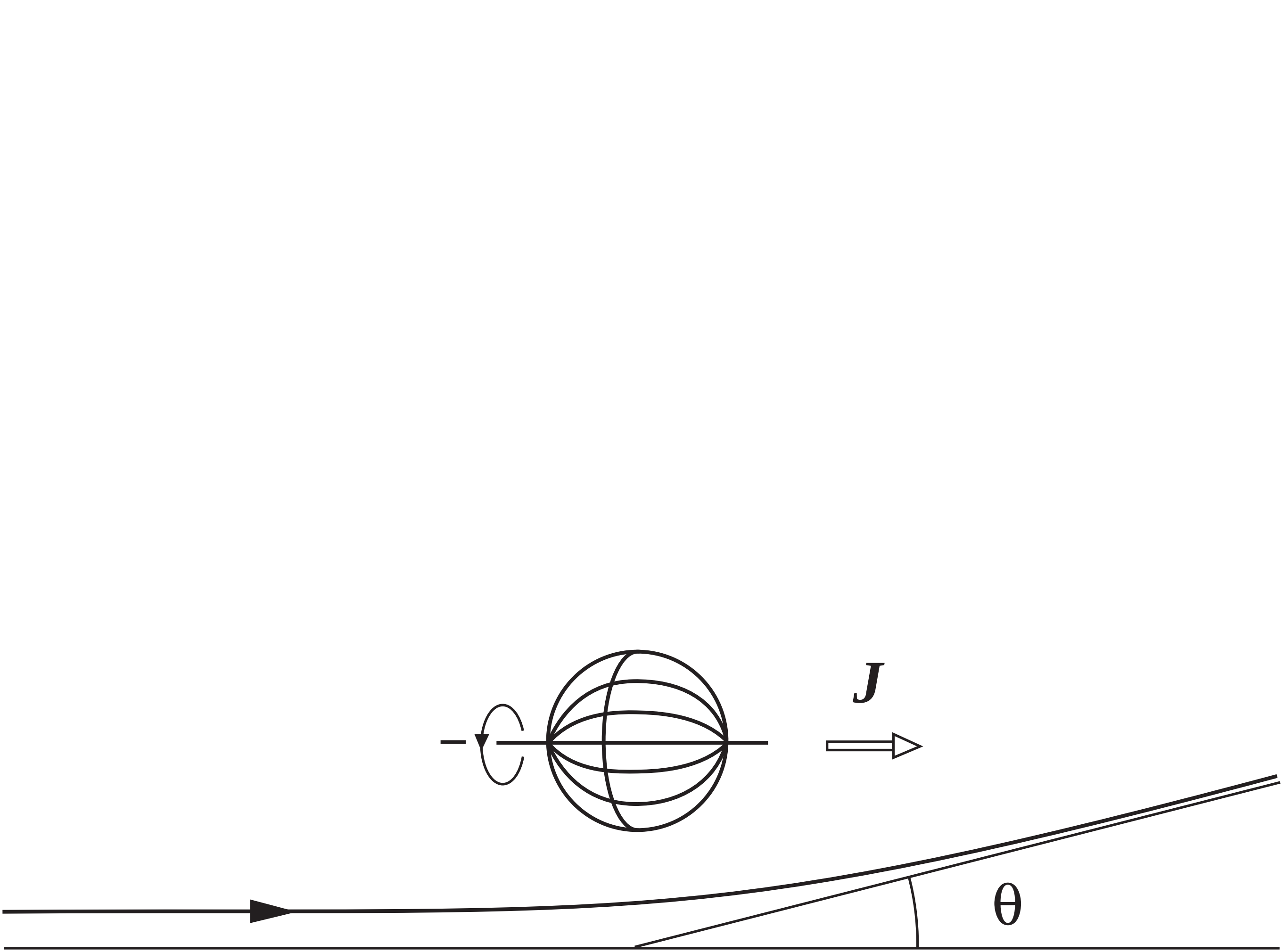

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